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**** SMALL ENTITY ****

Title

High safety file for root canal treatment

PROTECTING YOUR INVENTION OUTSIDE THE UNITED STATES

File of high safety for root canal treatment

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Abstract

Invention described here is devoted to the development of file of the high safety for root canal treatment. The principle of this device is based on the physics of torsion for the rod of arbitrary dependence of crosscut along the axis of rod. The theory has been developed in the frame of this project and it appears to be the expansion in theory of elasticity for torsion of rods.

Analysis has shown that existing files, having conical profile, possess of the high probability of separation at the very tip of the file, when it is stuck in narrow root canal. The theory developed for the rod of arbitrary profile allows calculate the shape of such instrument, which has no possibility of separation in canal even if it is overloaded by applied torque.

Manufacturing of files of this kind is very important because it makes dentists free of fear of breakdown while treating the root canal of higher complexity.

High Safety File for Root Canal Treatment

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The invention described here can be useful for constructing a number of devices including, but not limited to: High Safety File for Root Canal Treatment.

Description of Invention

1. Theory of elastic torsion of rod.

The principle of this new invention is based on the physics of elasticity of solid material, including our original theoretical analysis of torque transfer along the rod of instrument with arbitrary profile (dependence of crosscut characteristic dimension along the axis of rod). The analysis has shown that existing files, having conical profile [1], possess essential deficiency owing to their shape. The probability of fracture is the most at the very tip of these files, when they are getting stuck in the depth of root canal. The medical practice indicates that separation of files takes place mostly in the very depth of root canal, and the fraction is the thinnest part of the file, whose length is less or equal to a quarter of working part of the file or so [2, 7]. Below we will discuss the main physical causes of breakdown for conical metallic files, and the construction of the file of highest safety as well.

For this aim we analyze the torsion of thin rod with arbitrary dependence of its crosscut characteristic dimension on the coordinate z along the axis of the rod. Assume that the origin of coordinate system is at the tip of the rod, and the crosscut has symmetrical form (for example, circle, or right triangle, or square). Characteristic dimension could be the radius of circle, the side of right triangle, or the half-diagonal of square and so on. Let's suppose it is a certain function of *z*, for example *r(z)*. The problem consists in theoretical determination of the torsion angle per unit length as a function of *z*, i.e. we would like to find dependence

$$\tau(z) = \frac{d\varphi(z)}{dz} \quad , \tag{1}$$

where $\varphi(z)$ is the total angle of torsion of the crosscut located at the coordinate *z*. The expression (1) contains the increment of angle of torsion $d\varphi(z)$ between two crosscuts separated by the element *dz* along the axis of rod. This work consists in the generalization of the theory, which has been developed in [3] for torsion of homogeneous thin rod.

Let's assume torsional deformations to be small in order to the internal stresses would be always less than flow stress S_0 [4], i.e. they are always in the elastic range. For this problem as well as for the rod of uniform profile, when $r \neq f(z)$, such condition can be written in the form:

$$\Delta\varphi = \int_z^{z+r} \tau(z)\cdot dz \ll 1 \qquad \text{at any } z \quad , \tag{2}$$

where *r* is characteristic dimension of the rod, corresponding to coordinate *z*. It means that relative angle of torsion between two crosscut separated by a distance of the order of transverse dimension should be small at any *z*. We describe the torsion around the *z* axis, when the end at the point *z=0* is kept fixed, and the torque *M* is applied to the upper end of the rod at *z=l*, where *l* is the length of the rod, or the length of working part of the file. So the angle at the point *z* respectively to the plane, where the origin is located, is

$$\varphi(z)=\int_0^z \tau(z)dz \quad . \qquad (3)$$

The components of the displacement vector for the point, having coordinates *(x,y,z),* are as following:

$$u_x=\left[-\int_0^z \tau(z)dz\right]\cdot y=-\varphi(z)\cdot y \qquad \text{and}$$

$$u_y=\left[\int_0^z \tau(z)dz\right]\cdot x=\varphi(z)\cdot x \quad . \qquad (4)$$

The displacement along the axis *z* is described by the function $\Psi(x,y,z)$, which has been called the torsion function [3]:

$$u_z=\tau(z)\cdot\psi(x,y,z) \quad . \qquad (5)$$

So that each crosscut rotates around the *z* axis and becomes unplane surface. Components of deformation vector ***u*** allow us to find the components of the strain tensor, which are not equal to zero:

$$u_{xz}=\frac{1}{2}\tau(z)\cdot\left[\frac{\partial\psi}{\partial x}-y\right] \qquad \text{and}$$

$$u_{yz}=\frac{1}{2}\tau(z)\cdot\left[\frac{\partial\psi}{\partial y}+x\right] \quad . \qquad (6)$$

Other components are equal to zero, therefore the torsion is pure shear deformation. Components of the stress tensor can be written, using equations (6), as follows:

$$\sigma_{xz} = 2\mu \cdot u_{xz} = \mu\tau(z)\cdot\left[\frac{\partial\psi}{\partial x} - y\right] \qquad \text{and}$$

$$\sigma_{yz} = 2\mu \cdot u_{yz} = \mu\tau(z)\cdot\left[\frac{\partial\psi}{\partial y} + x\right] \qquad , \qquad (7)$$

where μ is the shear modulus or modulus of rigidity.

The next step is the derivation of equation for the torsion function from general equations, describing equilibrium state $\partial\sigma_{ik} / \partial x_k = 0$. In our case it can be written in the form:

$$\frac{\partial\sigma_{zx}}{\partial x} + \frac{\partial\sigma_{zy}}{\partial y} = 0 \qquad . \qquad (8)$$

The substitution of the stress tensor components (7) into equation (8) gives the differential equation for torsion function:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad . \qquad (9)$$

On the next step we will follow the method developed in [3] to provide the convenience of boundary conditions for their formulation. Let's introduce auxiliary function $\chi(x,y,z)$, which satisfies equations:

$$\sigma_{xz} = 2\mu\tau(z)\frac{\partial\chi}{\partial y} \qquad \text{and} \qquad \sigma_{yz} = -2\mu\tau(z)\frac{\partial\chi}{\partial x} \qquad . \qquad (10)$$

Comparison of the equations (10) to (7) allows us to write relation between functions ψ and χ:

$$\frac{\partial \psi}{\partial x} = y + 2\frac{\partial \chi}{\partial y} \qquad \text{and} \qquad \frac{\partial \psi}{\partial y} = -x - 2\frac{\partial \chi}{\partial x} \quad . \qquad (11)$$

It is worthy to mention that profile of the rod is not a sharp function of *z* (it means that modulus of *dr/dz* is not a very high value at any point), and the rod is thin (i.e. $r(z) \ll l$). Therefore the external forces, acting on the side surface, must be small compared to internal stresses in the rod. So we can put external forces to zero on the surface: $\sigma_{ik} n_k = 0$, where n_k is the component of the normal vector to the contour line of crosscut. Since the component n_z=0, this equation has the form:

$$\sigma_{zx} n_x + \sigma_{zy} n_y = 0 \quad . \qquad (12)$$

The substitution of the expressions (10) into (12) leads to the equation:

$$\frac{\partial \chi}{\partial y} n_x - \frac{\partial \chi}{\partial x} n_y = 0 \quad . \qquad (13)$$

The components of the normal vector to a plane contour, which appears the boundary of crosscut, are as follows:

$$n_x = -\frac{dy}{dl} \qquad \text{and} \qquad n_y = \frac{dx}{dl} \quad ,$$

where *x* and *y* are coordinates of points on the contour, and *dl* is an element of arc of the contour. So we have got from (13) the equation for χ:

$$\frac{\partial \chi}{\partial y} dy + \frac{\partial \chi}{\partial x} dx \equiv d\chi = 0 \quad . \qquad (14)$$

Since its total differential equals to zero, the function χ is constant on the line of contour. Let's return back to definitions (10) of this function: they contain only derivatives - and so that we can apply to χ any constant, including zero (if the crosscut is the singly connected surface):

$$\chi = 0 \tag{15}$$

on the line of contour of crosscut, corresponding to any point along axis z.

The torsion function Ψ is one-valued, so the integral of its differential $d\Psi$, taken around the closed contour must be zero. Using equations (11) we have for this integral:

$$\oint d\psi = \oint\left(\frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy\right) = -2\oint\left(\frac{\partial\chi}{\partial x}dy - \frac{\partial\chi}{\partial y}dx\right) - \oint(xdy - ydx) = 0 \quad ,$$

or, in general form:

$$\oint\frac{\partial\chi}{\partial n}dl = -S \quad . \tag{16}$$

Here $\partial\chi/\partial n$ is the derivative of the function χ along the outward normal to the line of contour, and S is the area enclosed by this line. In simplest cases S is the crosscut area of the rod, i.e. $\sim r^2$.

Let's determine the relations between physical parameters of the problem by means of free energy and elastic energy of the rod. The free energy per unit volume of the rod under torsion is:

$$F = \frac{1}{2}\sigma_{ik}u_{ik} = \sigma_{xz}u_{xz} + \sigma_{yz}u_{yz} = \frac{\sigma_{xz}^2 + \sigma_{yz}^2}{2\mu} \quad , \tag{17}$$

or, using expressions (10) for σ_{xz} and σ_{yz} ,we have:

$$F = 2\mu\cdot\tau(z)^2\cdot\left[\left(\frac{\partial\chi}{\partial x}\right)^2 + \left(\frac{\partial\chi}{\partial y}\right)^2\right] \quad . \tag{18}$$

The torsional elastic energy per unit length of the rod is the integral of *F* obtained over the crosscut area:

$$\int_S F ds = \frac{1}{2} C(z) \cdot \tau(z)^2 \quad , \qquad (19)$$

where

$$C(z) = 4\mu \int_{S(z)} \left[\left(\frac{\partial \chi}{\partial x} \right)^2 + \left(\frac{\partial \chi}{\partial y} \right)^2 \right] ds \qquad (20)$$

is so called torsional rigidity of the rod – in our case it is function of z since S and χ depend on z.

The total elastic energy of the rod is equal to integral from (19) taken along the length l of rod:

$$F_e = \frac{1}{2} \int_l C(z) \cdot \tau(z)^2 dz \quad . \qquad (21)$$

The torsional rigidity $C(z)$ is constant for uniform rod and its value depends only on modulus of shear μ of material, and on configuration and size of crosscut area.

In our case the rigidity is not constant, but a function of z due to $S(z)$ and $\chi(z)$. It has been shown in [3] that expression (20) for rigidity can be transformed to the expression containing two terms:

$$C(z) = 4\mu \oint \chi \frac{\partial \chi}{\partial n} dl + 4\mu \int_S \chi ds \quad . \qquad (22)$$

If the crosscut is singly connected, as it is often in practice, the first term vanishes owing to boundary condition χ=0 on the contour of crosscut. So that the rigidity is the second term:

$$C(z) = 4\mu \int_{S(z)} \chi(x, y, z) dx dy \quad . \tag{23}$$

The function (23) determines torsional rigidity of the rod, and allows us describe torsion angle per unit length *τ(z)* and the total angle of torsion $\varphi(z)$ along the axis of rod. Physics of torsional rigidity is working so that small value of torsion angle corresponds to the big value of rigidity, i.e. functions *C(z)* and *τ(z)* must be strictly connected.

This relation can be found by means of analysis of equilibrium state of the system under torsion. The torque *M* of pair of forces is applied to one end of the rod at *z=l,* and the counter end at *z=0* is kept stationary. The total energy of the system equals to the sum of elastic energy of the rod F_e and the potential energy *U* due to the action of the external forces:

$$F_{system} = F_e + U \quad . \tag{24}$$

Using equations (1) and (21), the total energy (24) can be written in the form:

$$F_{system} = \frac{1}{2} \int_l C(z) \cdot \left(\frac{d\varphi}{dz} \right)^2 dz + U \quad . \tag{25}$$

The variation of (25) respectively to the angle of torsion leads to the expression:

$$\int_l C(z) \frac{d\varphi}{dz} \frac{d\delta\varphi}{dz} dz + \delta U = 0 \quad . \tag{26}$$

Integration of (26) leads to the equation, which appears cardinal generalization of equation derived for uniform rod in [3]:

$$C(z)\cdot\tau(z)\cdot\delta\varphi+\delta U+\int_l\delta\varphi\left(\frac{dC}{dz}\cdot\tau+C\cdot\frac{d\tau}{dz}\right)dz=0 \quad . \qquad (27)$$

The value δU equals (with the sign minus) to the work done by pair of external forces when they made the torsion through the angle $\delta\varphi$:

$$\delta U=-M\cdot\delta\varphi \quad . \qquad (28)$$

Thus the expression (27) can be rewritten using (28) in final form:

$$\delta\varphi\cdot[C(z)\cdot\tau(z)-M]+\int_l\delta\varphi\left(\frac{dC}{dz}\cdot\tau+C\cdot\frac{d\tau}{dz}\right)dz=0 \quad . \qquad (29)$$

In this equation the variation $\delta\varphi$ is arbitrary value, so that both expressions in brackets should be equal to zero simultaneously.

First of them gives the equality of the product of torsion rigidity *C(z)* and torsion angle $\tau(z)$ to the value of torque applied to the rod – this is fundamental equation in the theory:

$$C(z)\cdot\tau(z)=M \quad . \qquad (30)$$

When uniform rod is under torsion, theory [3] gives the product of two constants. So we have derived general result for the torsion of rod, which has arbitrary profile *r(z)*, and uniform profile *r*=const is only particular case, when *C* and τ are constant too.

Second expression in brackets in (29) establishes relation between derivatives of these functions:

$$\frac{dC(z)}{dz}\cdot\tau(z)+C(z)\cdot\frac{d\tau(z)}{dz}=0 \qquad . \qquad (31)$$

From equation (31) it can be easily seen that relative increase of function *C(z)* equals to relative
decrease of function $\tau(z)$ on every element *dz*:

$$\frac{dC(z)}{C(z)}=-\frac{d\tau(z)}{\tau(z)} \qquad . \qquad (32)$$

The integration of expression (31) leads to the fundamental equation (30). Thus at the elastic torsion of the rod, having arbitrary profile, functions *C(z)* and $\tau(z)$ so correspond each other that their product at any crosscut (i.e. at any coordinate *z*) is equal to applied torque *M*.

It means that for parts of the rod, where the torsion rigidity is high (for example, rod is thick), the torsion angle per unit length should be small, and vice versa. In other words, the most critical parts of the rod have low torsion rigidity, and namely they are dangerous respectively to the breakdown at the large angles of torsion. The critical shear stress will be reached first of all in the part of the rod where angle of torsion per unit length $\tau(z)$ passes through an absolute maximum or has a highest value.

It is needful to mention that parts of the rod where the torsional rigidity decreases sharply can be also critical. If the derivative *dC(z)/dz* has a large modulus, the magnitude of derivative $d\tau(z)/dz$ could occur very high. Actually, from equations (31) and (30) one can see that derivative of the torsion angle is

$$\frac{d\tau(z)}{dz}=-\frac{M}{C(z)^2}\cdot\frac{dC(z)}{dz} \qquad . \qquad (33)$$

This expression shows that at the point *z*, where magnitude *C(z)* is small, the modulus of right part of (33) could be very high. The total angle of torsion between two crosscuts separated along axis by the element *dz* can be written in this case as the sum of two terms of the series:

$$d\varphi = \tau(z)\cdot dz + \frac{1}{2}\frac{d\tau(z)}{dz}\cdot dz^2 \qquad . \qquad (34)$$

So that namely the second term may bring in decisive supplement when $d\varphi$ exceeds critical value.

Thus in such parts of the rod the shear stresses achieve their critical values earlier than in parts where profile *r(z)* and torsional rigidity vary smoothly along the axis *z*.

Engineering of instruments should be done taking into account both of these features of theory of elastic torsion.

Let's analyze two practically important cases of existing instruments under torsion:

1.1. Assume that radius of crosscut varies linearly along the axis of round conical file – this shape is close to Flex Master's shape:

$$r(z) = \frac{1}{2}(D_0 + k\cdot z) \qquad , \qquad (35)$$

where D_0 is diameter of file tip at z=0, and *k* is the slope of linear function. So the shape of the file is the cut cone with linear generatrix and nearly round crosscut. The tip of the file is kept immovable, and to the end of working part at $z=l$ the torque *M* is applied. The auxiliary function for such rod is [8]:

$$\chi(x,y,z)=\frac{1}{4}\left[r(z)^2-x^2-y^2\right] \quad , \qquad (36)$$

and torsion rigidity has the form:

$$C(z)=\frac{\pi\mu\cdot r(z)^4}{2} \quad . \qquad (37)$$

The torsion angle per unit length of the rod is corresponding to equation (30):

$$\tau(z)=\frac{M}{C(z)}=\frac{2M}{\pi\mu\cdot r(z)^4} \quad . \qquad (38)$$

The expression (38) shows the fast decrease of torsion angle with the increase of radius *r(z)* along the rod as the hyperbolic function $\sim r(z)^{-4}$. So the torsion is concentrated near to thin tip of the file, and that is why the critical angle of torsion and critical stress will be reached here at the tip, when overloading takes place. The medical practice and engineering research [2,5] confirm this theoretical conclusion. At the larger values of *z* and *r(z)* the function $\tau(z)$ varies slowly and far from critical magnitude.

Fig.1 shows the profile *r(z)* for NiTi file, having parameters as following: D_0= 0.25 mm, *k*=0.02 (2% cone), *l*=16 mm, $\mu=4\cdot10^{11}$ dyne/cm^2.

In Fig.2 the rigidity *C(z)* of this file is represented as the function along the working part of instrument. The fast increase of torsional rigidity along the axis $\sim r(z)^4$ can be seen.

Fig.3 represents the torsion angle per unit length as the function of *z*, calculated for two values of applied torque: *M*=100 dyne·cm and M_1=150 dyne·cm. The torsion angle at the thin tip is much higher than in the middle of file and in the last third of its working length: $\tau(0)/\tau(l)=30$.

Thus, when the overload occurs, the breakdown is more likely to be at the first quarter of working length near to its thin tip. Analysis of experimental data shows namely similar dependence [5].

1.2. Assume now that file has right triangle form of crosscut (for example, RaCe file [6]). Characteristic dimension *a* of this triangle is the length of its side, and it is linear function of *z*. The axis *x* is one of the height of triangle. So we have for this case:

$$a(z) = a_0 + k \cdot z \quad , \tag{39}$$

where a_0 is the side of triangle at the tip *z*=0. The torsional rigidity is again [8] the strong power function of *z*:

$$C(z) = \frac{\sqrt{3}}{80} \cdot \mu \cdot (a_0 + k \cdot z)^4 \quad . \tag{40}$$

The dependence of torsion angle $\tau(z)$ will be hyperbolic function reciprocal to *C(z)*:

$$\tau(z) = \frac{80M}{\sqrt{3} \cdot \mu} \cdot \frac{1}{(a_0 + k \cdot z)^4} \quad . \tag{41}$$

This function is represented in Fig.4. It demonstrates the highest values at the tip of file and dramatic changes in the first quarter of working length. The probability of breakdown again localized in the thinnest part of file – it means that breakdown will take place in the depth of root canal if file is overloaded, and that is the most objectionable case for practitioners.

Thus, the theory of elasticity for torsion of rod, having arbitrary profile along its axis, allows us fulfill quantitative analysis of torsion angle for files, which are currently in practice. Analysis shows dangerous distribution of torsion angle and points out the part of working length of the instrument where breakdown is the most probable. This part is the quarter of file at the very tip for instruments, having conical profile.

Further, the theory induces ideas for engineering of instrument with profile, which assures higher safety. This issue will be discussed at the end of this text.

2. Instability of rod.

The elastic rod under torsion appears to be an interesting example, demonstrating elastic instability when critical angle of torsion is reached. This phenomenon has been analyzed first mathematically by Leonard Euler. Since that this analysis took its place in theoretical physics [see, for example, 8]. Here we will utilize the results of this analysis with comments necessary to apply them correctly.

It is clear, that state of the rod after the loss of stability must be described by equations for strong bending. However the critical parameters on the threshold could be obtained from equations for small deviations from stable state, i.e. for slight bending. At the critical loading (respectively to the stability, not yet to breakdown) the straight form of the rod under torsion corresponds to a certain equilibrium state within which slight deviation to the bending exists.

Assume that round rod is under torsion and boundary conditions are close to ones when file is getting stuck in root canal: the tip is steady kept and to the upper end the torque of critical value M_c is applied. Let's determine the critical magnitudes of torsion angle τ_c and hence torque M_c, when straight form of rod becomes unstable.

For this aim we will describe at fist bending of simplest rod. Assume that rod has round crosscut of radius $r=const$ and major inertial moments of crosscut are $I_1=I_2=I$, then the torque of forces applied to the rod can be written in the form:

$$\vec{M} = E \cdot I \cdot \left[\vec{t} \, \frac{d\vec{t}}{dl} \right] + C \cdot \tau \cdot \vec{t} \qquad , \qquad (42)$$

where E is Young's modulus of material, $\vec{t}$ is the unit vector directed along the tangent to the rod, which is seeing now as the elastic line, and the derivative $d\vec{t}/dl$ is the vector of curvature of elastic line and its modulus equals to $1/R_c$ (the curvature of bending in spiral – see below). The torsion angle here is equal to constant. The full system of equations, describing the equilibrium of bended rod is:

$$\frac{d\vec{F}}{dl} = -\vec{K} \qquad \text{and}$$

$$\frac{d\vec{M}}{dl} = \left[\vec{F}\vec{t} \right] \qquad . \qquad (43)$$

First of them means that resultant of forces acting on the element dl (it is bended dz) of the rod is equal to zero, where $\vec{K}$ is the external force per unit length. Second equation is obtained from the equivalency to zero of the total torque of forces applied to this element. This second equation together with equation (42) gives us the expression describing the equilibrium of such rod:

$$E \cdot I \left[\vec{t} \, \frac{d^2\vec{t}}{dl^2} \right] + C \cdot \tau \cdot \frac{d\vec{t}}{dl} - \left[\vec{F}\vec{t} \right] = 0 \qquad . \qquad (44)$$

Since we are describing the slight bending, the vector $\vec{t} = \vec{t}_0$ is constant and directed along the axis of rod. More over, since there is not of external forces along the rod, the derivative

$$\frac{d\vec{F}}{dl} = 0 \qquad . \qquad (45)$$

Taking this simplification into account, after differentiation of equation (44) it can be led to the form:

$$E \cdot I\left[\vec{t}_0 \frac{d^3\vec{t}}{dl^3}\right] + C \cdot \tau \cdot \frac{d^2\vec{t}}{dl^2} = 0 \qquad . \qquad (46)$$

This equation is the main one in the theory of instability of rods. It can be reformed, and this procedure discussed in details in [8]. We will cite the final form of it here:

$$\xi'''' - i\kappa\xi''' = 0 \qquad , \qquad (47)$$

where $\kappa = C\tau / EI$, and the symbol ' means differentiation by z.

We must find the solution of this equation, which satisfies following conditions: $\xi = 0; \xi' = 0$ at z=0 and z=l. Suppose that solution has the form:

$$\xi = a \cdot \left(1 + i\kappa z - e^{i\kappa z}\right) + bz^2 \qquad . \qquad (48)$$

The system of equations for coefficients a and b leads to solution:

$$e^{i\kappa l} = \frac{2 + i\kappa l}{2 - i\kappa l} \qquad , \qquad (49)$$

which means that κ should satisfy the equation:

$$\frac{\kappa l}{2} = tg\frac{\kappa l}{2} \qquad . \qquad (50)$$

The minimal root of the equation (50) is:

$$\frac{\kappa d}{2} = 4.49 \qquad . \qquad (51)$$

Returning back to initial parameters of the problem, we can write for critical value of the torsion angle per unit length the following expression:

$$\tau_c = \frac{8.98 \cdot E \cdot I}{C \cdot l} \qquad . \qquad (52)$$

The critical value of torque M_c can be obtained from fundamental equation (30) using (52):

$$M_c = C \cdot \tau_c = \frac{8.98 \cdot E \cdot I}{l} \qquad . \qquad (53)$$

Expressions (52) and (53) will be utilized to get estimates in engineering calculations for files of round crosscut.

We can also consider boundary conditions with ends of file kept by joints, so they can turn a little bit at points of keeping around x and y axes. In this case the equation for coefficients a, b takes the form:

$$\xi = a\left(1 - e^{i\kappa z} - \frac{\kappa^2}{2} \cdot z^2\right) + bz \qquad . \qquad (54)$$

Parameter κ is determined by the condition, which can be expressed in form: $e^{i\kappa d} = 1$, or $\kappa \cdot l = 2\pi$.

In terms of initial parameters this leads to expression:

$$\tau_c = \frac{2\pi \cdot E \cdot I}{C \cdot l} \quad . \qquad (55)$$

The comparison (55) to (53) shows that they give the same order of magnitude for critical torsion angle per unit length τ_c.

These solutions allow us also to carry out the estimate for critical angle and critical torque, which could be applied to the file not only of cylindrical but of conical profile as well. In this last case it is necessary to make a choice of crosscut for which the estimate is calculated.

In the part 1 of this text it has been shown that $\tau(z)$ reaches its largest value at the tip of instrument, where *r* of crosscut is a small value. Therefore in the same part the critical value will be reached first of all, and namely this thinnest part of file will experience first the instability, bending and conversion to the spiral. In other words, namely for this part of the file the most probable is getting stuck in the narrow root canal, since the radius of spiral R_s is always larger than radius of file and of canal *r*.

3. Radius of bending and radius of spiral.

Let's derive the radius of bending R_c, when the stability of file is lost. The free energy per unit volume is described by left part of equation (17), and since in the case of pure bending the only component σ_{zz} of stress tensor is not equal to zero, it can be written using parameters *E, x,* and R_c in the form:

$$\frac{1}{2}\sigma_{ik}u_{ik} = \frac{1}{2}\sigma_{zz}u_{zz} = \frac{1}{2}E \cdot \frac{x^2}{R_c^2} \quad . \qquad (56)$$

Integration of expression (56) over crosscut area *S* leads to the free energy per unit length for bended rod:

$$\frac{E}{2R_c^2}\int_S x^2 ds \quad . \tag{57}$$

In part 1 of this text we have determined *R* as the radius of curvature of neutral surface, here we discuss bending of thin rod, for which *r«R,* and therefore radius is converted into R_c, which is the radius of bending of the rod treated here as the elastic line, so that $1/R_c^2$ is outside of integral in expression (57). If we introduce the inertia moment of crosscut respectively to the axis *y* as

$$I_y = \int_S x^2 ds \quad , \tag{58}$$

then free energy per unit length for rod bended around axis *y* is:

$$\frac{E}{2R_c^2}\cdot I_y \quad . \tag{59}$$

Now we must introduce the moment of forces, acting on given crosscut (this moment is called bending moment). To the element *ds* of crosscut surface the force $\sigma_{zz}\cdot ds = (x/\ R_c)E\cdot ds$ is applied.

This force is directed along axis *z*, and its moment respectively to axis *y* is $x\sigma_{zz}ds$. So the total moment, or torque respectively to this axis is:

$$M_y = \frac{E}{R_c}\int_S x^2 ds = \frac{E}{R_c}\cdot I_y \quad . \tag{60}$$

Thus, the curvature $1/R_c$ of bending is proportional to the acting at the crosscut bending moment and inversely proportional to the Young's modulus of material and to the moment of inertia of crosscut:

$$\frac{1}{R_c} = \frac{M_y}{E \cdot I_y} \quad . \qquad (61)$$

Now we know the free energy per unit length of rod for bending (59) and for torsion (19), and this allows us to explore the deformation in general case. It is a matter that at bending strong enough (but tensor of deformation is still a small value, so the process is in elastic range), simultaneously the torsion takes place. So the deformation is the pure bending and pure torsion at the same time. This situation is described in details in theory of elasticity [8]. Here we will explore the final expression for elastic free energy of bended rod, which is written as the integral taken along the length of rod:

$$F_e = \int_l \left(\frac{I_1 \cdot E}{2} \Omega_\xi^2 + \frac{I_2 \cdot E}{2} \Omega_\eta^2 + \frac{C}{2} \Omega_\zeta^2 \right) dl \quad , \qquad (62)$$

where I_1, I_2 are major moments of inertia of crosscut, C is rigidity of the rod respectively to the torsion, and Ω_ξ, Ω_η, and Ω_ζ appear to be components of deformation vector $\vec{\Omega} = d\vec{\varphi}/dl$, which determines "velocity" of rotation of coordinate systems connected to the crosscuts along the length of the rod. For example, at the pure torsion respectively to the axis z the component Ω_ζ occurs to be equal to the torsion angle per unit length τ (see expression (21)). In pure bending respectively to the axis ξ, or η instead of Ω_ξ, or Ω_η ensues the value $1/R_c$ (as it was in the expression (59)).

Thus, the elastic free energy of round rod can be written in the form:

$$F_{er} = \int_l \left(\frac{IE}{2R_c^2} + \frac{C\tau^2}{2} \right) dl \quad . \qquad (63)$$

At the pure torsion the energy is concentrated in elastic torsion of the rod, so in the expression (63) the only second term exists, and the first term is absent (rod is straight and the radius of curvature R_c is equal to infinity). The interesting situation appears to be, from the instability point of view, when the torsion angle reaches and exceeds the critical value τ_c. When the stability is lost, the part of elastic energy of torsion should be converted into the energy of bending. So the rod will possess the shape of spiral with radius of curvature R_c and the radius of spiral R_s (it is, for example, radius of imaginary cylinder on which the rod is wound). The radius of curvature can be estimated in following manner. Assume that the process of conversion of elastic energy of torsion to the elastic energy of bending is going on until they are equal to each other, i.e.:

$$\frac{IE}{2R_c^2} = \frac{1}{2} \cdot \frac{C\tau_c^2}{2} \quad . \qquad (64)$$

Equation (64) gives the radius of curvature of bending:

$$R_c = \sqrt{\frac{2IE}{C\tau_c^2}} \quad . \qquad (65)$$

The spiral, having radius of bending R_c , is wound on the cylinder of radius R_s, or on the cone of radius $R_s(z)$, when under torsion is the rod of conical profile similar to one described in part 1.1.

The radius of the spiral R_s is always less than radius of bending R_c, and for practical estimates could be assumed as a half of it:

$$R_s \approx 0.5R_c \quad . \qquad (66)$$

It is important to mention, that radius of spiral is larger than radius of canal, which made by means of file of radius *r*. It means that spiral would be getting stuck in canal, if the instability occurred.

Thus, when working at the motor driven mode, the critical angle of torsion per unit length τ_c should be under control, and it means that torque applied to the instrument should be less than critical value: $M_c = C(z)\cdot\tau_c(z)$. This value is much less than torque of breakdown.

It is clear also, that critical value must be determined for the weakest part of the file. In proposed construction we especially provide such a place (groove), where file could be wound in spiral and broken when overloaded, and it is located outside the canal near to the handle of file. So that even if the breakdown occurred, the file could be extracted out of canal easily. For this aim we provide special head near to this groove to catch the file when extracting it from canal, or to keep the file immovable when groove or body of file is under test and calibration.

Let's return back to engineering. We have seen earlier that files of conical profile have the highest torsion angle per unit length near to their tips, where characteristic dimension is small. Here in this part the critical value τ_c will be reached first of all, and the spiral will be getting stuck in narrow canal. Namely that feature can be used as explanation of the fact that fragments of conical files, which were broken in canal, are less in their size than first quarter of working length.

3.1. Let's calculate critical angle of torsion and critical torque for stainless steel file with cylindrical profile.

Parameters of problem are following: E=2.0·10^{12} dyne/cm^2; r=0.2mm; l=20mm; $I = \pi\cdot r^4/\ 4$;

$C = \mu\pi\cdot r^4/\ 2$; $\mu = E/\ 2(1+\nu)$; $\nu = 0.3$.

The radius of bending is

$$R_c = \left(\frac{2IE}{C\tau_c^2}\right)^{1/2} = \frac{E\pi \cdot r^4}{M_c 2\sqrt{2(1+\nu)}} \quad . \qquad (67)$$

The critical angle of torsion per unit length is:

$$\tau_c = \frac{8.98 \cdot EI}{Cl} = \frac{8.98(1+\nu)}{l} = 5.8 \, {rad}/{cm} \quad .$$

So almost full revolution per centimeter we have before the spiral will appear, i.e. for file of 2 cm length ~ 2 revolutions.

The critical torque is

$$M_c = C \cdot \tau_c = 2.24 \frac{\pi \cdot r^4 E}{l} = 1.12 \cdot 10^6 \, dyne \cdot cm = 11.2 N \cdot cm \, .$$

Using equation (67) we can now calculate radius of bending when the stability of the rod is lost:

$$R_c = 0.28 cm \quad .$$

So the radius of spiral is of the order of 1.4 mm, and this is much larger than radius of the canal, which is of the order of 0.2 mm. The file will get stuck in narrow canal inevitably, if the spiral is formed.

3.2. Let's calculate critical parameters for stainless steel conical file.

Parameters of problem are as follows: k=0.02; r_0=0.5D_0=0.1 mm; l=2.0 cm; E=2.0·10^{12} dyne/cm^2;

$r(z) = 0.5(D_0 + k \cdot z)$.

Now radius of curvature of bending will be dependent on coordinate z:

$$R_c(z)=\left(\frac{2I(z)E}{C(z)\tau_c(z)^2}\right)^{1/2}=\frac{E\pi\cdot r(z)^4}{M_c 2\sqrt{2(1+\nu)}} \quad . \qquad (68)$$

In equation (68) we have written $\tau_c(z)$ as a function of *z*, however it appears to be dependent only on Poisson's ratio ν and the length of file *l*:

$$\tau_c=\frac{8.98(1+\nu)}{l} \quad .$$

It is very interesting result. Now critical torque is determined only by rigidity *C(z)*, and will be reached at first in the part, where rigidity has a minimal value, i.e. at the tip of file:

$$M_c=C(0)\cdot\tau_c=2.24\frac{\pi\cdot r_0^4 E}{l}=7\cdot 10^4\,dyne\cdot cm=0.7N\cdot cm \quad .$$

Thus, the critical torque is on one order of magnitude less than in first example.
The radius of bending of critical part is:

$$R_c(0)=\frac{E\pi\cdot r_0^4}{M_c 2\sqrt{2(1+\nu)}}=0.28cm=2.8mm \quad .$$

The radius of spiral is ~1.4 mm, and it is much larger than radius of canal~0.1mm in critical part. When applied torque is more than critical value 0.7 N·cm, file will be broken at its thinnest part.
These results will be used now to formulate principles of engineering of high safety file and to calculate the most important parameters of new construction. All of them are the subject of patent.

4. High safety file. Main features of engineering and construction of file.

1. The profile $r(z)$ should provide the uniformity of torsional rigidity along the working part of the instrument, i.e. $r(z)$=const, and so $C(z)$=const, and $\tau(z)$=const. So that there is no any extremum points in the distribution of torsion angle per unit length on this part of the file, and there is no points or regions where probability of break has greatest value. A certain smooth maximum can exist only in the function $C(z)$ in the region of cutting head ($0<z<z_{ch}$, see Fig.5), since its diameter should be a little bit larger than diameter of the body of file.

2. The profile $r(z)$ should provide the smooth shape of the file to prevent from the appearance of high value derivative $|dr(z)/dz|$, and so $d\tau(z)/dz$. The sharp change of the profile is dangerous in particular in that place, where $C(z)$ reaches its smallest value.

3. The body of the file in its working length should not have any thread, except the thread on the cutting head, and must be polished to prevent from friction and getting stuck in narrow canal in working regime, or in particular in the case of instability of the rod, when it is overloaded.

4. To calculate precisely the permitted torque, which could be applied to the instrument, the theory of instability of the rod under torsion must be employed for the body of working part of file (equation 53). Theory gives critical value of the torque M_c, which is very important parameter of the instrument, and it will be used further in our engineering calculation. Namely this number should be used by practitioner on the torque control device, when file is used in motor driven mode.

5. The construction of the file should provide the possibility of experimental measurement of critical torque. For this aim the special node should be provided above the end of working part (at the coordinate $z>l$). For example, it could be hexagonal head, like the head of bolt (between z_{sh} and $z=n$, see Fig.5), to apply the calibrating torque to the working part, while the cutting head is kept fixed. If

critical torque is measured, its value should be written out into certificate of the file together with calculated value for the usage in practice of the least of them.

6. Higher of the node along the z-axis (at the $n < z < h$, see Fig.5) the special groove should be turned to provide the weakest part of the rod respectively to the torsion. Namely at this part of the instrument the separation should take place in the case of inaccurate employment or accidental overloading. This groove is always located outside the canal in the process of treatment, and even if the breakdown occurred, the file could be extracted from the canal as the whole piece, using the node to take it and rotate in opposite direction (node was described in section 5). The special extractor, or even usual tweezers could be helpful. The shape of groove (for example, toroidal) should be designed corresponding to section 2, and the fracture of the thinnest part at the bottom of groove must come when the torque applied to the instrument is by 10 to 20% larger than critical value M_c, which has been found in section 4. The radius of thinnest part r_c at the bottom of groove (neck) should be determined by experimental measurements on special series of grooves of different radius r turned of the same material and by the same process as the final product. In this measurement left part of the groove is kept fixed by the node (section 5) and to the right part the torque M should be applied. By the series of measurements the torques, at which the failure has been observed, could be obtained. So the characteristic $M_f = f(r)$ can be plotted. The similar calibration plot should be measured for each type of instrument and for each type of process. The critical radius r_c corresponds to the point at which the line $M=1.2M_c$ intercepts with calibration curve (see Fig.6). The statistics should be used to get a good accuracy, which must be of the same order of magnitude, as the best commercial torque control device could provide.

Fig.5 represents the shape of the high safety file, satisfying to principles of design described above:

- The shape of file is cylindrical (not a conical file [1] or inverted taper [9]) to provide uniformity of the torsion angle per unit length along the working part of the instrument.

- The cylinder **B** of its body has not any thread or cutting edges but it is electro-chemically polished [9] to avoid the friction and getting stuck. Lifting of rasping is carried out by the rise of instrument and washing off.

- The cutting head **A** could be of different kind from cutting smoothly and working as the path finder in anatomic canal (for thin files) to cutting sharply and aggressively (for more thick files). Threads of the head should provide lifting up of rasping.

- The node **C** with hexagonal head is smoothly converted into the body **B** on the left and into the groove **D** on the right. It serves for rotation and extraction of the file in the case of failure, for measurements of critical torque, and for calibration process of the groove neck strength.

- The groove ends by the transition into the handle, which must fit to the standard engine designed for root canal treatment and have precise control of applied torque and speed of rotation. These parameters should be specified for each instrument of this kind.

Fig.6. represents the procedure of determination of critical radius at the neck of the groove from the experimental dependence of torque of failure on the radius of neck.

Thus, the theory of elasticity of torsion allows us to formulate principles of engineering and design for the files of high safety for root canal treatment. It could be successfully applied also for engineering of instruments and machine details of different kind where the precision of torque transfer, the weight and size of construction are the matter of great concern. All applications are the subject of this patent.

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Captions

Fig.1. The profile *r(z)* of NiTi conical file with round crosscut.

Fig.2. The torsional rigidity *C(z)* of conical NiTi file with round crosscut.

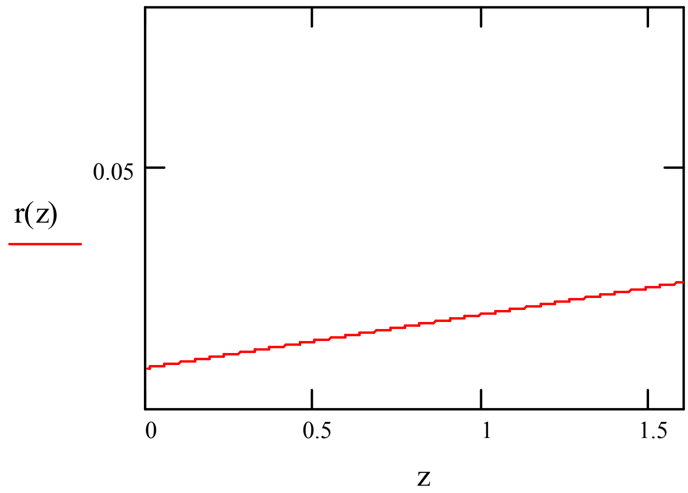
Fig.3. Torsion angle per unit length $\tau(z)$ of conical NiTi file with round crosscut. Torque is parameter.

Fig.4. Torsion angle per unit length $\tau(z)$ of RaCe conical file with triangle crosscut. Torque is parameter.

Fig.5. The qualitative representation of construction of high safety file and the distribution of characteristic dimension of profile *r(z)*, torsional rigidity *C(z)*, and torsion angle per unit length $\tau(z)$ along the axis of file.

Fig.6. Determination of critical radius r_c of the neck of groove using the calibration curve.

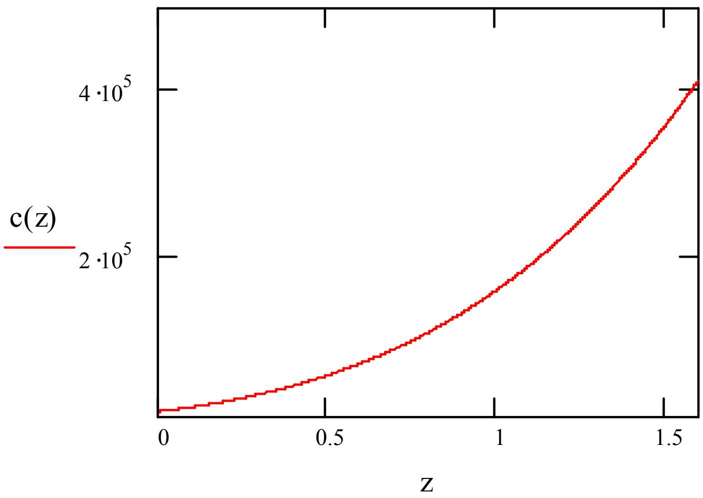
Fig. 1



$$r(z) := \frac{1}{2}(0.025 + 0.02 \cdot z)$$

Radius of file, cm

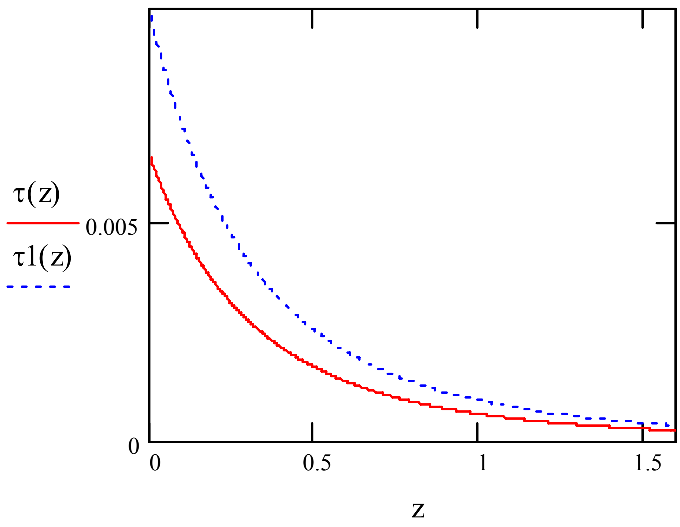
Fig. 2



$$c(z) := \frac{1}{2} 3.14 \cdot 4 \cdot 10^{11} r(z)^4$$

Torsional rigidity ,dyne·cm^2

Fig. 3



$\tau(z)$ ......M := 100 dyne · cm

$\tau 1(z)$ ......M := 150 dyne · cm

$$\tau(z) := \frac{2 \cdot 100}{3.14 \cdot 4 \cdot 10^{11} \cdot r(z)^4}$$

$$\tau 1(z) := \frac{2 \cdot 150}{3.14 \cdot 4 \cdot 10^{11} \cdot r(z)^4}$$

Torsion angle per unit length, $\frac{rad}{cm}$

Conical NiTi file.

$M = 100$ dyne·cm

$l = 1.6$ cm

$k = 0.02$

$\mu = 4 \cdot 10^{11} \frac{dyne}{cm^2}$

Theory has done for conical rod of 1.6 cm length with k=0.02 and D(0)=0.025cm.
NiTi alloy is carachterized by shear modulus μ.
This modulus should be found in literature fo NiTi alloy.

$r(z) := \frac{1}{2}(D + k \cdot z)$ where z is the axis along rod from thin tip, and D is diameter at z=0.

$c(z) := \frac{1}{2}\pi \cdot \mu \cdot r(z)^4$ Rigidity is very strong function of radius r(z).

$\tau(z) := 2\frac{M}{\pi \cdot \mu \cdot r(z)^4}$ Torsion angle per unit length is reciprocal function of c(z).

M is torque applied at the point z=1.6cm: M= 100and 150 dyne*cm.

It is clearly seen from Fig.3 for τ(z) and τ1(z) that increasing of torque M loads mostly the thinnest part of the file, where torsional rigidity c(z) is the weakest one.

Fig. 4

NiTi file of triangle cross section with its side a=0.02 cm at z=0, and k=0.02 as obliquity.

$k := 0.02$

$\mu := 4 \cdot 10^{11}$

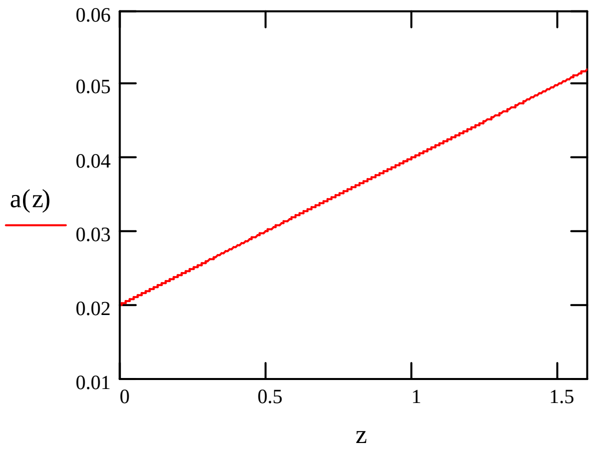
$M := 100 dyne \cdot cm$

$M1 := 150 dyne \cdot cm$

Side of triangle as a function of z.

$$a(z) := a(0) + k \cdot z$$

$$a(z) := 0.02 + 0.02 \cdot z$$

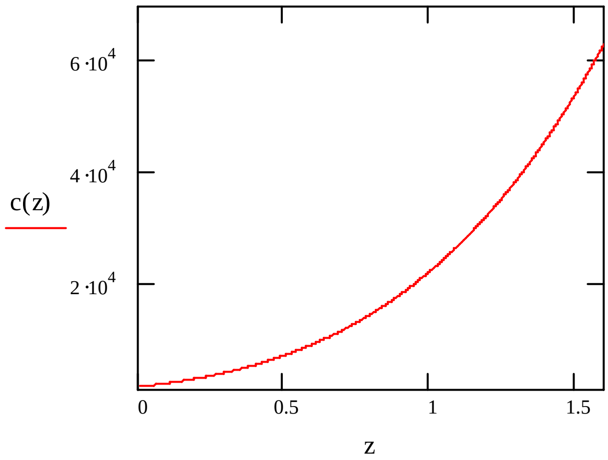


Side of triangle crosscut, cm

Torsional rigidity of the file

$$c(z) := \frac{3^{\frac{1}{2}}}{80} \cdot \mu \cdot (a(0) + k \cdot z)^4$$

$$c(z) := \frac{3^{\frac{1}{2}}}{80} \cdot 4 \cdot 10^{11} \cdot a(z)^4$$



Torsional rigidity, dyne·cm^2

Torsion angle per unit length

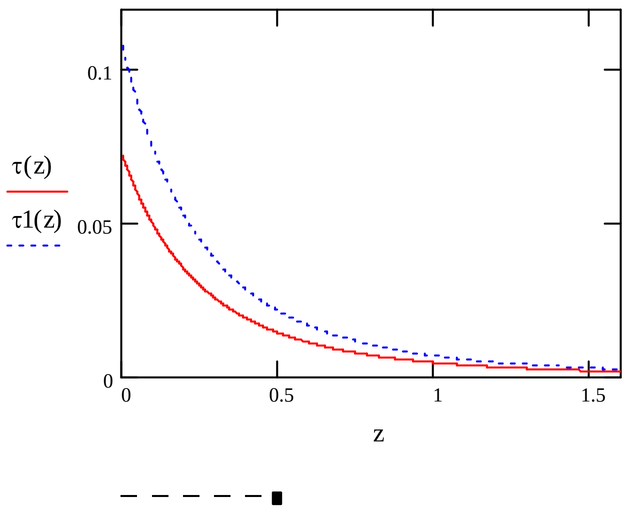
$$\tau(z) := \frac{80 \cdot M}{3^{\frac{1}{2}} \cdot \mu \cdot a(z)^4}$$

$$\tau(z) := \frac{80 \cdot 100}{3^{\frac{1}{2}} \cdot 4 \cdot 10^{11} \cdot a(z)^4}$$

$$\tau1(z) := \frac{80 \cdot 150}{3^{\frac{1}{2}} \cdot 4 \cdot 10^{11} \cdot a(z)^4}$$

$$\tau(z) := \frac{80 \cdot 100}{3^{\frac{1}{2}} \cdot 4 \cdot 10^{11} \cdot \left[(0.02 + 0.02 \cdot z)^4\right]}$$

$$\tau1(z) := \frac{80 \cdot 150}{3^{\frac{1}{2}} \cdot 4 \cdot 10^{11} \cdot (0.02 + 0.02 \cdot z)^4}$$



The region of the most probable breakdown

Torsion angle per unit length for two values of torque: M and M1. $\frac{rad}{cm}$

Fig. 5

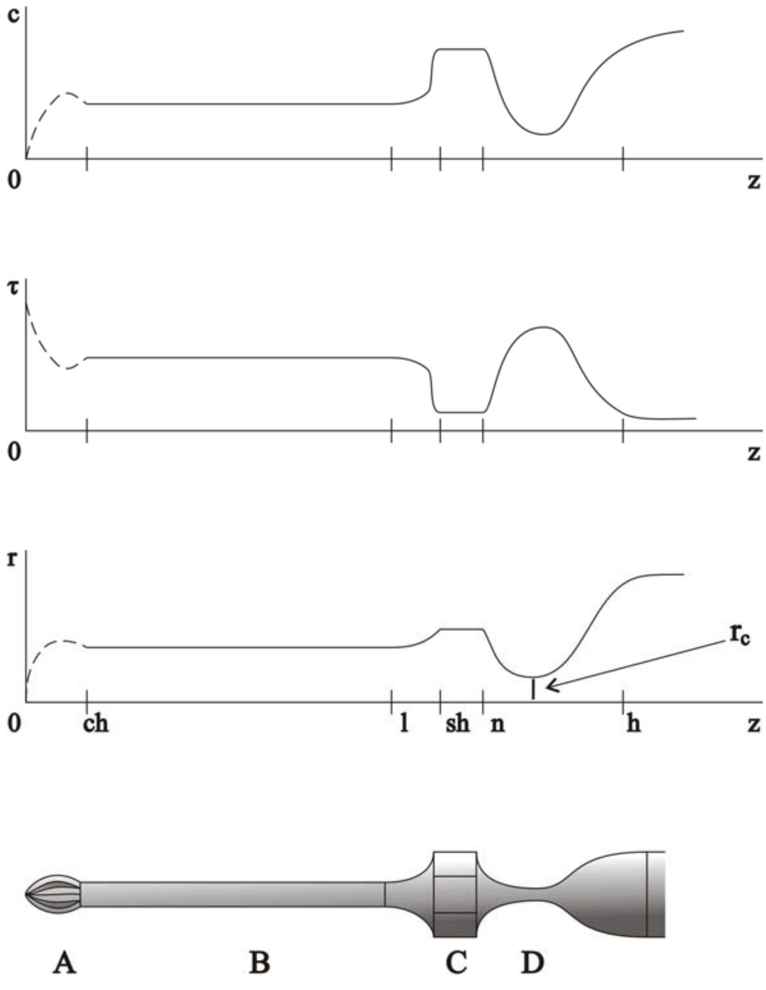


Fig. 6